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Labor Supply Flexibility, Taxation and Cross-Border Portfolio Choice

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Abstract

This paper investigates the interaction between labor supply flexibility, taxation and cross-border portfolio choice to explain two defining features of the financial globalization. First, despite financial integration, investors still hold a disproportionate ratio of local equities, the so-called home bias in portfolio holdings. The second feature is an asymmetric composition of international portfolio where the USs deficits in net foreign asset position have been driven by the foreigners large accumulation of its low yield safe assets while on the assets side the US has increased its holdings of foreign high yield equity and FDI.

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1 Introduction

The last two decades have witnessed an unprecedented pace of financial globalization and integration that carries many important implications for cross-border macroeconomic interactions arised from international transmissions of shocks and changes in policies and environment among countries. First, as seminally documented by Lane and Milesi-Ferretti (2001, 2007), since around 1980s there is an explosion in gross external positions in a two-way integration where countries hold a substantial amount of assets abroad and at the same time owning large liabilities to foreigners. Gross asset and liability positions [1] in many countries have increased to the levels equal to 200-300 percent of their gross domestic product (GDP) and are allocated across a various types of asset such as equities, foreign direct investment and debts. Second, net capital flows also increased significantly to a level and in the direction [2] that existing theoretical frameworks such as the intertemporal approach and traditional saving-investment decision theories are inable to explain. Third, financial globalization and integration have also caused financial interdependence and contagion in which financial crises are strongly and rapidly transmitted across countries via various channels such as trade linkages and capital flows. [3]

This research project focuses on two notable observations in the context of recent financial globalization and integration. First, the US's net foreign asset (NFA) position has deteriorated significantly but as pointed out by Ob-

[1] Gross assets of a country are the total value of claims by its residents to the rest of the world while gross liabilities are claims by all foreigners to that country. Net foreign assets are the diference between gross assets and gross liabilities.

[2] The traditional theory implies capital tends to flow from capital-rich to capital-poor while recent empirical evidence shows that capital flows from emerging economies to the US and UK, causing the phenomenon of global imbalances.

[3] Two remarkable episodes are the Asian 1997-1998 financial crisis and the recent 2007-09 global financial crisis.

stfeld and Rogoff (2000), Gourinchas and Rey (2007) and others, the large deficits of the US position have been mainly driven by the foreigners' large accumulation of US's low yield safe assets such as treasuries and government bonds while on the assets side the US has increased its holdings of foreign high yield equity assets and foreign direct investment (FDI).[4] Consequently, even moderate movements in assets prices or exchange rates may cause significant fluctuations in the countries net foreign assets position because they are applied to large amounts of gross cross-border holdings. This empirical evidence has, therefore, ignited active discussions about the effects of capital gains on foreign equity position or the so-called valuation effects, and their role on the sustainability of current account and the external adjustment process, especially question of whether the US could reduce its deficits through valuation effects.[5] Second, despite these unprecedented paces of financial integrations and rise in gross external positions, it is surprising that investors still do not diversify their portfolio away from national assets but still hold a disproportionate fraction of local equities or the so-called home bias in equities. In 2007, US investors held more than 80 percent of domestic equities and the home bias in equities is documented in almost all developed countries.

Motivated by the aforementioned empirical facts, this research project investigates the interaction between labor supply flexibility, taxation and

[4]The value of US's gross assets increased from 30% of GDP in 1982 to a peak of 124% in 2007 before falling after the start of the recent global financial crisis.

[5]In general, the overall valuation effects consists of valuation effects from asset prices, valuation effects from exchange rates and other valuation effects from statistical revisions and emissions. Since a large ratio of US's gross assets are denominated in foreign currencies while a large fraction of its gross liabilities is in USD, a depreciation of the USD since 2000s generated valuation effects from exchange rates for the US. The US's overall valuation effects reached the peak of 35% of its GDP in 2007, in which valuation effects from asset prices accounted for roughly a half of the total effects.

cross-border portfolio choice. It aims to explain home bias in equity and difference in the composition of international portfolio holdings and then study the valuation effects and external adjustment.

In particular, I incorporate two ideas into the research project. The first one explores the idea that the flexibility in varying labor supply ex post, which acts as a kind of insurance against negative investment shocks, generates a more flexible investment opportunity choice hence may encourage people to assume more risk-taking in portfolio investment ex ante. The second idea examines two competing views on the endogeneity between taxation on equity income and risk-taking. The first view, which seems to be more intuitive, is that taxation on risky assets reduces risk-taking. The second view that taxation increases risk-taking is, however, a bit more counter-intuitive. It states that a hike in the tax rate by decreasing the return to the asset and raising the marginal utility of income will eventually increase the demand for higher return asset and encourage more risk-taking decisions.

It is needed to stress that the above two ideas about the relationship between labor supply flexibility and risk-taking and between taxation and risk-taking is not new. However, as later mentioned in the literature review and methodology, they are either explored separately or/and more importantly applied in a closed macro-economy context. The novelty of this research project is to incorporate both the two ideas into an open economy macroeconomic model in order to explain home bias and asymmetric compositions of international portfolio holdings.

In the meantime, this research project, in terms of methodology, extends the existing literature by marrying the two-country international macroeconomic framework with the continuous time stochastic general equilibrium analysis that incorporates capital accumulation, labor supply flexibility and endogenous portfolio choice. The marriage enables the model to address home bias and asymmetric composition of international portfolio holdings from a new point of view.

This paper is organized as follows. Section 2 reviews related literature and methodology and at the same time elaborates methodology and key ideas of this research. Section 3 describes detailed economic setting of the two-country macroeconomic finance model with portfolio choice and flexible labor supply and then analyzes macroeconomic equilibrium with several specifications in taxation rules and their implication. Section 5 concludes.

2 Methodology

2.1 Literature Review

The recent process of financial globalization and integration has drawn considerable attentions in both academics and policymaker circles into trying to understand its causes and consequences. Unfortunately, most of earlier open economy macro models are not well suited for analyzing many interesting features related to the process such as global imbalances, home bias and asymmetric compositions of portfolio holdings, etc... because they dont consider equity markets and portfolio choice. On top of that, it is a technical challenge to incorporate endogenous portfolio choice into the traditional workhorse of macroeconomic models because these models are usually solved and then simulated by using first order linearization.[6] Portfolio choices are, however, also determined by risk structure of various assets, which are influenced by higher order of shocks and volatilities.

As a result, recent international macroeconomic literature that incorporates endogenous cross-border portfolio choice has been advanced in two different directions. The first approach is to consider approximation meth-

[6]This strand of macroeconomic literature is closely related and derived from the real business cycle literature pioneered by Kydland and Prescott (1982).

ods of higher order around the deterministic steady state for the traditional Dynamic Stochastic General Equilibrium (DSGE) two-country open macroeconomy models such as the perturbation-based local method developed Devereux and Sutherland (2010, 2011) and Tille and van Wincoop (2010). In particular, the method developed in these papers combine higher order approximation of the portfolio problem with other existing analysis the non-portfolio of the models; it first derives constant portfolios by using the second order approximation with one asset then use the constant portfolio to obtain a third order approximation, which is in turn utilized to derive a first order approximation to the portfolio choice. The method of Devereux and Sutherland (2010, 2011) and Tille and van Wincoop (2010) utilize a set of algorithms to compute the first and second order approximations to solutions of DSGE models such as Schmitt-Grohe and Uribe (2004)[7] and its underlying mathematical theory is related to the Bifurcaton Theorem and Implicit Function Theorem originally developed by Judd (1998).

There have been, however, controversial debates about this local method of whether it can correctly capture the behaviors of the economy models when the underlying shocks are large and the economy models are far from the deterministic steady state. For example, Rabitsch et al., (2015) shows that although this method performs relatively well when shocks are small at the business cycle frequencies it may lead to inaccurate results at long horizons in asymmetric settings. One of the reasons is that the method of Devereux and Sutherland (2010, 2011) and Tille and van Wincoop (2010) critically depends on the NFA value point that is guessed at the beginning of the approximation process. Rabitsch et al., (2015) then proposes for a global solution method that is more technical and computationally demanding.

By contrast, the second approach that aims to find exact solutions by assuming several specifications and simplifications and solves models analyt-

[7]See the Appendix A for a brief introduction of the perturbation method.

ically without approximations. The main advantage of this second approach is apparently that the economy model with portfolio choice can be analyzed beyond the steady states but its disadvantage is that the exact solutions only can be found under specific conditions. This approach is developed by Helpman and Razin (1978), Cole and Obstfeld (1991) and recently extended by Pavlova and Rigobon (2007, 2010).

2.2 Methodology

The methodology adopted in this research project is in the line with the second approach. However, unlike Helpman and Razin (1978), Cole and Obstfeld (1991), Pavlova and Rigobon (2007, 2010) and others that consider only an endowment economy model, I incorporate production with both capital accumulation and labor in order to study the interaction between taxation, labor supply flexibility and cross-border portfolio choice.

As a result, the economy model deployed here is related to the continuous-time stochastic dynamic general equilibrium model seminally developed by Eaton (1981).[8] It then follows Bodie et.al. (1992) to incorporate labor leisure choice to address the interaction between labor supply flexibility and risk-taking.[9] On the other hand, the relationship between taxation, risk-taking

[8]Grinols and Turnovsky (1994) and Obstfeld (1994) use similar continuous stochastic genenal equilibrium growth model to address open macroeconomic issues like exchange rate determination and asset prices and international risk-sharing and growth.The framework is further extended to analyze the effects of risk on equilibrium growth in open developing economies that face imperfect world capital market in Turnovsky and Chattopadhyay (2003).

[9]Basak (1999) also develops a tractable continuous time general model of a representative consumer-laborer-investor in the same line with Bodie et.al. (1992) to study the relation between investor's labor supply, consumption behaviors, financial wealth and human capital. However, the model of Basak (1999) has no capital and the analysis is only comparative static.

and economic growth in a small open economy is discussed in Asea and Turnovsky (1998). To address the question of how taxation on capital income affects household portfolio choice and growth, in particular the amount of risk taken and investment level, Asea and Turnovsky construct a stochastic small open economy model, which incorporates both capital accumulation and taxes on domestic and foreign capital and bonds. In contrast to conventional wisdom, their paper shows that higher taxes induce more risk-taking. Kenc (2004) extends Asea and Turnovsky (1998) in a closed economy model with endogenous labor-leisure choice to investigate the effects of labor income on capital investment and the effects of taxes on labor income on growth, which is probably the closest one to my methodology in this aspect.

Nonetheless, none of these papers are suitable to analyze global imbalances, home bias and composition of cross-border portfolio choice since they are either a closed or small open economy model. Therefore, in terms of methodology, this research project extends the existing literature by marrying the two-country international macroeconomic framework with the continuous time stochastic general equilibrium analysis that incorporates capital accumulation, labor supply flexibility and endogenous portfolio choice.

In the meantime, one of the challenges in building and solving dynamics macroeconomic models with portfolio choice and/or a financial sector in continuous time is the applications of relatively complicated mathematics: stochastic differential equations and stochastic optimal control that have not been familiar with many economists.[10] This methodology is closely related to the field of continuous-time finance seminally developed Robert Merton (1992) and have been widely used in recent the macro-finance nexus literature. [11]

[10]The Appendix B for a very brief introduction of stochastic optimal control in continuous time.

[11]For example, Brunnermeier and Sanikov (2014) construct a continuous time macro finance model that goes beyond log-linearization to study global dynamics with pretty

3 The Model

Consider a two-country, one-good model with representative agents and their respective governments. Variables of the foreign economy are starred and h refers to the holding of domestic residents while f refers to holdings of foreign agents. In outlining model, we will focus on describing the home economy.

3.1 The Economic Setup

3.1.1 Production Technology

We consider a continuous-time world economy model with production in a finite horizon, $[0, T]$ with uncertainty representated by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ on which two-dimensional Brownian motion $\overline{y(t)} = (y(t), y^*(t)', t \in [0, T]$. All stochastic processes are adapted to $\{\mathcal{F}_t\}, t \in [0, T]$

There are a continuum of competitive firms distributed uniformly on the unit interval $[0, 1]$, where each firm has access to the same technology with constant returns to scale production function as in the spirit of Arrow-Romer growth model:

$$dY_i(t) = [\nu dt + \eta dy(t)] K_i(t)^{1-\alpha} N_i(t)^{\alpha} \tag{3.1}$$

where dY_i is the flow of output of firm i, $K(t)$ and $N(t)$ are capital and labor efficiency inputs, $dy(t)$ is the increment to a zero mean and unit variance Brownian process and ν and η denote the drift and deviation of productivity shocks. And, as in Romer (1986), the labor efficiency units are defined as the product of labor and the average economy-wide stock of capital, $K(t)$, which each individual firm takes as given but which in equilibrium accumulates

tractable solutions to study the highly nonlinear amplification effects and endogenous risk phenomena. The paper demonstrates that large aggregate fluctuations and crises can happen even for a level of exogenous risk.

endogenously from private capital stock:

$$N_i(t) = L_i(t)K(t) \tag{3.2}$$

By defining:

$$\mu = \nu[N_i]^\alpha \tag{3.3}$$

$$\sigma = \eta[N_i]^\alpha \tag{3.4}$$

and submitting into the production (3.1) and note that at equilibrium labor units are the same amongs firms, we can obtain an AK production function with a stochastic linear coefficient as:

$$dY_i(t) = [\mu dt + \sigma_K dy(t)]K_i(t) \tag{3.5}$$

Hence, the rate of returns on home capital, dR_K over the period $(t, t+dt)$ will follow the following Ito process:

$$dR_K(t) = r_K dt + \sigma_K dt \tag{3.6}$$

where $\sigma_K = (1-\alpha)\sigma$ and r_K is the expected marginal capital production evaluated at the equilibrium level of capital and labor $\bar{K}, \bar{L}$

$$r_K = E\left(\frac{\partial[dY_i(t)]}{\partial K_i(t)}\right)_{K=\bar{K}, L=\bar{L}} = (1-\alpha)\mu \tag{3.7}$$

The wage rate is assumed to be known at the beginning of time t and is set equal to the expected marginal production of labor over the period $(t, t+dt)$ as:

$$w = E\left(\frac{\partial[dY_i(t)]}{\partial N_i(t)}\right)_{K=\bar{K}, L=\bar{L}} \tag{3.8}$$

The total returns to labor, dR_L, over the period $(t, t+dt)$ is therefore equal to $dR_L = wdt$. With this specification, the wage rate is fixed over the period $(t, t+dt)$ while all short-run fluctuations in output resulted from stochastic return to capital.

Similarly, the production technology side of the foreign economy can be expressed as follows:

$$dY_i^*(t) = [\mu dt + \sigma_K dy^*(t)]K_i^*(t) \tag{3.9}$$

$$dR_K^*(t) = r_K^* dt + \sigma_K dt \tag{3.10}$$

$$dR_L^* = w^* dt \tag{3.11}$$

where:

$$r_K^* = E\left(\frac{\partial[dY_i^*(t)]}{\partial K_i^*(t)}\right)_{K^*=\bar{K^*}, L^*=\bar{L^*}} = (1-\alpha)\mu \tag{3.12}$$

$$w^* = E\left(\frac{\partial[dY_i^*(t)]}{\partial N_i^*(t)}\right)_{K^*=\bar{K^*}, L^*=\bar{L^*}} \tag{3.13}$$

3.1.2 Households

The representative household can hold internationally traded bonds issued by other households, B and equity, K (financial assets) and processes one type of non-tradable human capital, K. Each period, the household supplies labor, $L(t)$, at the real wage rate, w, and obtains financial income from holding assets so its budget constraint is given as follows:

$$W = p_B B + K + p_H H \tag{3.14}$$

where p_B, p_H are the prices of bonds and human capital asset, respectively and the price of equity is denominated to one. The household then spreads its income on two type of consumption goods: home produced goods $C_H(t)$ and foreign produced goods $C_F(t)$ and financial investments by holding bonds and equity.

In particular, we assume that the domestic representative household maximizes its expected lifetime utility defined over home consumption goods

$(C_H(t))$ and foreign consumption goods $C_F(t)$, and leisure $(l(t))$ over the period $[0, T]$ as follows:

$$E_0 \int_{t=0}^{T} e^{-\rho t} \theta_H(t) \left[\chi \left(a_H log(C_H(t) + (1-a_H) log(C_F(t)))\right) + (1-\chi)\left(l(t)\right)\right] dt \tag{3.15}$$

This form of utility function ensures that at the equilibrium the consumption-wealth ratio remains constant and the household's allocation of time between leisure and labor is also constant. The home representative household's utility is maximized subject to the following stochastic wealth accumulation equation:

$$dW = W[b(t)dR_B + (1-b(t))dR_K] - (C_H + p_F C_F)dt - wl(t)dt \tag{3.16}$$

where $b(t)$ is the portfolio share of bonds in the household's financial wealth and dR_i is the stochastic after-tax rate of return on asset i, $i = B, K$ and p_F is price of foreign produced goods or the real exchange rate in this two-country model.

And similarly, the foreign representative household maximizes its expected lifetime utility as follows:

$$E_0 \int_{t=0}^{T} e^{-\rho t} \theta_F(t) \left[\chi \left(a_F log(C_F^*(t) + (1-a_F) log(C_H^*(t)))\right) + (1-\chi)\left(l^*(t)\right)\right] dt \tag{3.17}$$

subject to the following wealth accumulation equation:

$$dW^* = W^*[b^*(t)dR_B^* + (1-b^*(t))dR_K^*] - (C_H^* + p_F C_F^*)dt - w^* l^*(t)dt \tag{3.18}$$

where in both the above utility functions, χ is the weight of consumption goods against leisure and ρ is the time preference, and a_H, a_F are the weights on the home goods of each country, which can be used to express the home-bias in consumption by imposing the condition that $a_H > a_F$. Finally, the demand shocks are expressed by $\theta_H(t), \theta_F(t)$, which are martingale and arbitrary adapted stochastic processes, with $\theta_H(0) = 1$ and $\theta_F(0) = 1$.

3.1.3 The government sector

The government in each country chooses its expenditure and financing it by proportional taxation on both capital and labor income. For simplicity, we assume that government expenditure does not yield direct utility for the household sector. The tax revenue, T, of the domestic government evolves as follows:

$$dT = [\bar{\tau}_K(1-\alpha)\mu + \tau_L \alpha\mu]Kdt + [\tau_k \sigma_K]Kdy \tag{3.19}$$

where τ denotes the tax rate and tax rates can be different between deterministic and stochastic components of capital and labor income as in Turnovsky (1997, 2000) and the rates on the deterministic components are denoted by a bar.

Government expenditure dG is assumed to be proportional to national income as follows:

$$dG = \bar{g}\mu Kdt + g\sigma Kdy \tag{3.20}$$

and as taxation, the proportions of government expenditure to income are different between the between deterministic and stochastic components and the proportion on the deterministic components are denoted by a bar.

Following Turnovsky (1997, 2000), we assume that the government continually balances its budget so that its expenditure is always equal to its tax revenue so it does not issue government bonds. That is:

$$dG = dT \tag{3.21}$$

The foreign government sector can be similarly expressed as follows:

$$dT^* = [\bar{\tau^*}^*_K(1-\alpha)\mu + \tau^*_L\alpha\mu]K^*dt + [\tau^*_k\sigma_K]K^*dy^* \tag{3.22}$$

$$dG^* = \bar{g^*}\mu K^*dt + g\sigma K dy^* \tag{3.23}$$

$$dG^* = dT^* \tag{3.24}$$

Note that, we allow different tax rates among home and foreign countries in order to analyze the impact of different fiscal policy on the portfolio choice and capital accumulation.

3.2 Macroeconomic Equilibrium and Implications

Since we assume that the government sector does not issue bonds, at equilibrium the level of bonds issued by the private sector in this two-country model should be equal to zero, hence:

$$B(t) + B^*(t) = 0 \tag{3.25}$$

As a result, at equilibrium the world's total wealth equates to the amount of total equity in the domestic and foreign countries as follows:

$$W(t) + W^*(t) = K(t) + K^*(t) \tag{3.26}$$

The world's goods market equilibrium condition is therefore:

$$dY(t)+dY^*(t) = dC_H(t)+dC_F(t)+dC^*_H(t)+dC^*_F(t)+dK(t)+dK^*(t) \tag{3.27}$$

In the next step, we will solve for the optimal portfolio choice and optimal growth path of the model and impose various fiscal (taxation) rules to study their effects on the equilibrium.

[To be added]

4 Concluding remarks

This paper constructs an international macroeconomic finance model with labor supply flexibility, taxation and cross-border portfolio choice to study two defining features of the financial globalization: the home bias phenomenon in portfolio holdings the an asymmetric composition of international portfolio where the USs deficits in net foreign asset position have been driven by the foreigners large accumulation of its low yield safe assets while on the assets side the US has increased its holdings of foreign high yield equity and FDI.

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Appendix A: Perturbation Method

The key idea of the perturbation method can be described by three steps:

1. Transform the original problem into a perturbation problem indexed by a small perturbation parameter in such a way that the zeroth-order approximation has analytical solution.

2. Index the solution as a function of the parameter and solve the new problem for a particular choice of the perturbation parameter.

3. Use the previous analytical solution to approximate the solution of the original problem.

In particular, the set of optimality conditions of a DSGE economy model can be expressed as follows:[12]

$$E_t\{F(Y_{t+1}, Y_t, X_{t+1}, X_t)\} = 0 \tag{4.28}$$

E_t is the mathematical expectation operator conditional on information available at time t, Y_t is the vector of non-predetermined variables, and $X_t = [x_t^1, x_t^2]'$ is the state variable vector, x_t^1 is endogenous predetermined state variables while x_t^2 is exogenous state variables. Particularly, x_t^2 follows exogenous process given as:

$$x_{t+1}^2 = \Lambda x_t^2 + \tilde{\eta}\sigma\epsilon_{t+1} \tag{4.29}$$

where $\tilde{\eta}, \sigma$ are given parameter. The solution of the optimal plan is of the form:

$$Y_t = g(X_t, \sigma) \tag{4.30}$$

$$X_{t+1} = h(X_t, \sigma) + \eta\sigma\epsilon_{t+1} \tag{4.31}$$

[12]For more details, see Schmitt-Grohe and Uribe (2004)

where $\eta = [\emptyset, \tilde{\eta}]'$, these equations describe the policy and transition functions respectively. The key point here is that the policy and transition functions are expressed as functions of perturbation parameter σ.

First, we are interested in the non-stochastic steady state of the DSGE model, which is defined as the following conditions:

$$F(\bar{Y}, \bar{Y}, \bar{X}, \bar{X}) = 0 \tag{4.32}$$

It is apparent that $\bar{Y} = g(\bar{X}, 0)$ and $\bar{X} = h(\bar{X}, 0)$

To get the first-order approximation of the solution, use first-order Taylor series expansion of the policy and transition functions around the steady state:

$$Y_t = g(\bar{X}, 0) + g_x(\bar{X}, 0)(X - \bar{X}) + g_\sigma(\bar{X}, 0)\sigma \tag{4.33}$$

$$X_{t+1} = h(\bar{X}, 0) + h_x(\bar{X}, 0)(X - \bar{X}) + h_\sigma(\bar{X}, 0)\sigma \tag{4.34}$$

In order to compute partial derivative coefficients $g_x, g_\sigma, h_x, h_\sigma$ we substitute the solution of policy and transition functions into the the set of optimality conditions of a DSGE economy model:

$$F(X, \sigma) = E_t\{F(g(h(X, \sigma) + \eta\sigma\epsilon', \sigma), g(X, \sigma), h(X, \sigma) + \eta\sigma\epsilon', X)\} = 0 \tag{4.35}$$

Because the above equation is an identity, all derivatives $F_{X^i,\sigma^j} = 0$. From this, we can obtain partial derivative coefficients $g_x, g_\sigma, h_x, h_\sigma$.

The steps to compute partial derivatives coefficients for the second-order approximation of the solution will be similar as above.

Appendix B: Stochastic Optimal Control in Continuous Time

It is well-known that there are two major ways in solving optimal control problems: the Dynamic Programming (DP) method and the Pontryagin's maximum principle method. However, the former is preferred in the stochastic cases because the latter requires the solution of a stochastic differential equation (SDE) with a terminal condition rather than an initial condition, which is unsuitable under uncertainty (see more for example Fleming and Stein). Unfortunately, in the continuous-time context the lack of smoothness of the value function even for simple optimal problems put a considerable limitation to the applicability of the DP method. More specifically, under the classical approach or the so-called verification procedure/theorem, one starts by finding a classical solution of the associated Hamilton-Jacobi-Bellman (HJB) equation and then verifies that it is equal to the value function of the original control problem. A major drawback to the verification procedure is that it needs to assume a sufficiently smooth solution of the HJB equation, which in many cases is not satisfied. Moreover, the verification procedure is an indirect method because it starts with the associated HJB equation so unable to take full advantage of the structure of the underlying control problem in order to find a solution.

As a result, in order to avoid the smoothness problem of the value function in using the DP method, Brunnermeier and Sanikov (2014) has to assume linear preference and fixed labor supply for both households and experts in their model. These simplifications imply that consumption is proportional to total wealth and the value function is just equal to total net-worth, hence enabling the model to have tractable solutions. However, they are not well supported by empirical evidence, making the model not very suitable for quantitative work.

A breakthrough in the DP approach is the theory of viscosity solutions seminally developed by Crandall and Lions (1983) and Lions (1983). Under this approach, one first establishes a dynamic programming principle (DPP) originated from an optimal control problem and then use this DPP to prove that the value function is a solution in a weaker sense, or the so-called viscosity solution of the associated HJB equation. The next step to show that this value function is the unique viscosity solution of the HJB equation among a certain class of functions, therefore establishing that the HJB equation completely characterizes the value function so can be used to find an optimal control. The major advantage of the viscosity solution approach is that it does not require the smoothness of the value function, hence can be a very powerful method for solving much more general optimal control problems, where the value function is not smooth and the verification procedure does not work (see for example Fleming and Soner, 2006). Moreover, it has been pointed out that there is no general framework for this approach and each optimality problem will require specific solution, hence can potentially providing challenging but promising harvest for further research in macro finance nexus models.